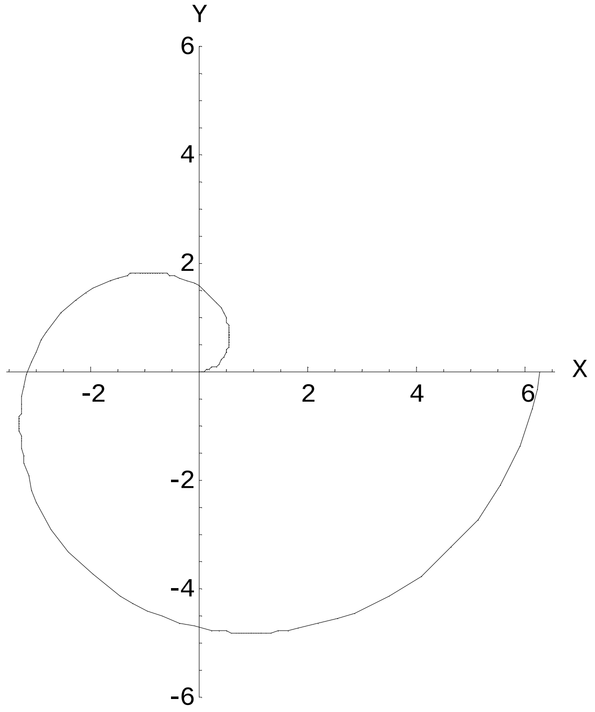
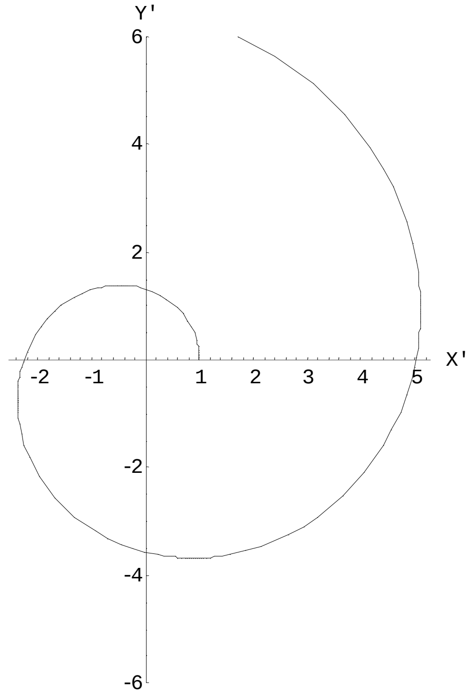
Oplossing: voorbeeld oefening

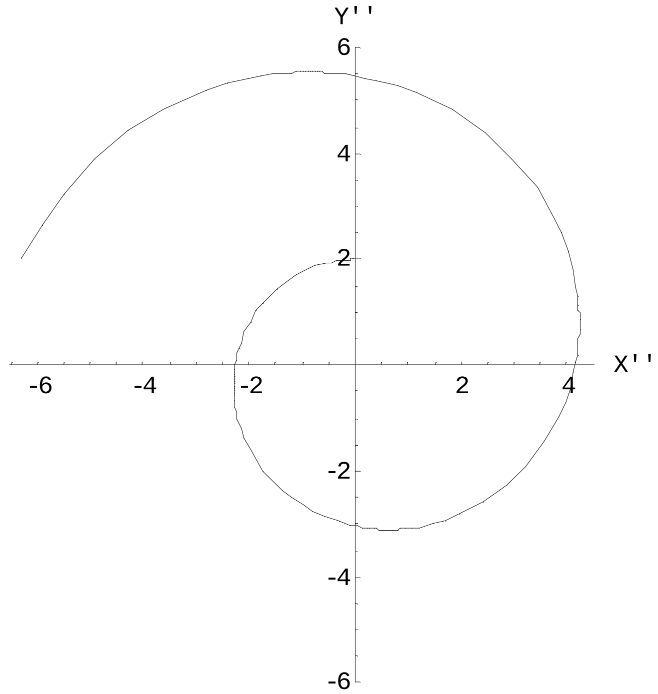
De absolute baan : $\begin{cases} X = v_0 t \cos \omega t \\ Y = v_0 t \sin \omega t \end{cases}$

De absolute snelheid : $\begin{cases} X = v_0 \cos \omega t - v_0 t\, \omega \sin \omega t \\ Y = v_0 \sin \omega t + v_0 t\, \omega \cos \omega t \end{cases}$

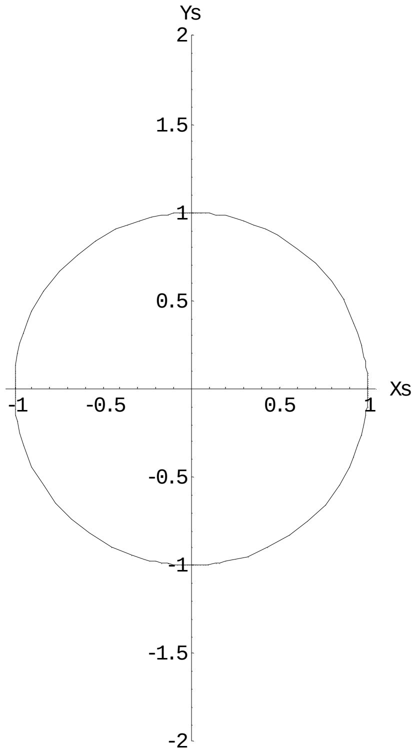


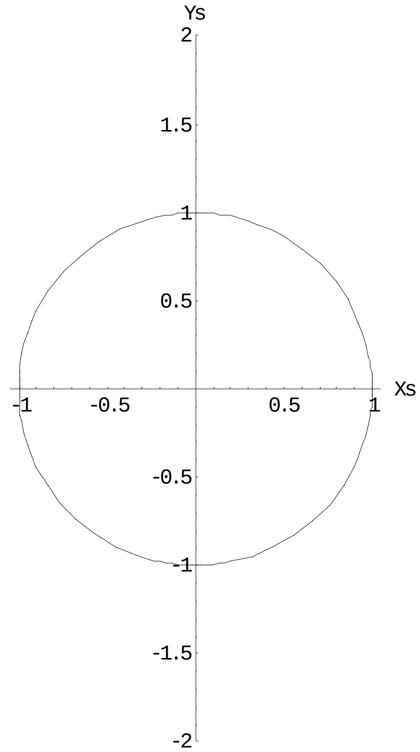


De absolute versnelling : $\begin{cases} X = -v_0 \omega \sin \omega t - v_0\, \omega \sin \omega t - v_0 t\, \omega^2 \cos \omega t \\ Y = v_0 \omega \cos \omega t + v_0\, \omega \cos \omega t - v_0 t\, \omega^2 \sin \omega t \end{cases}$

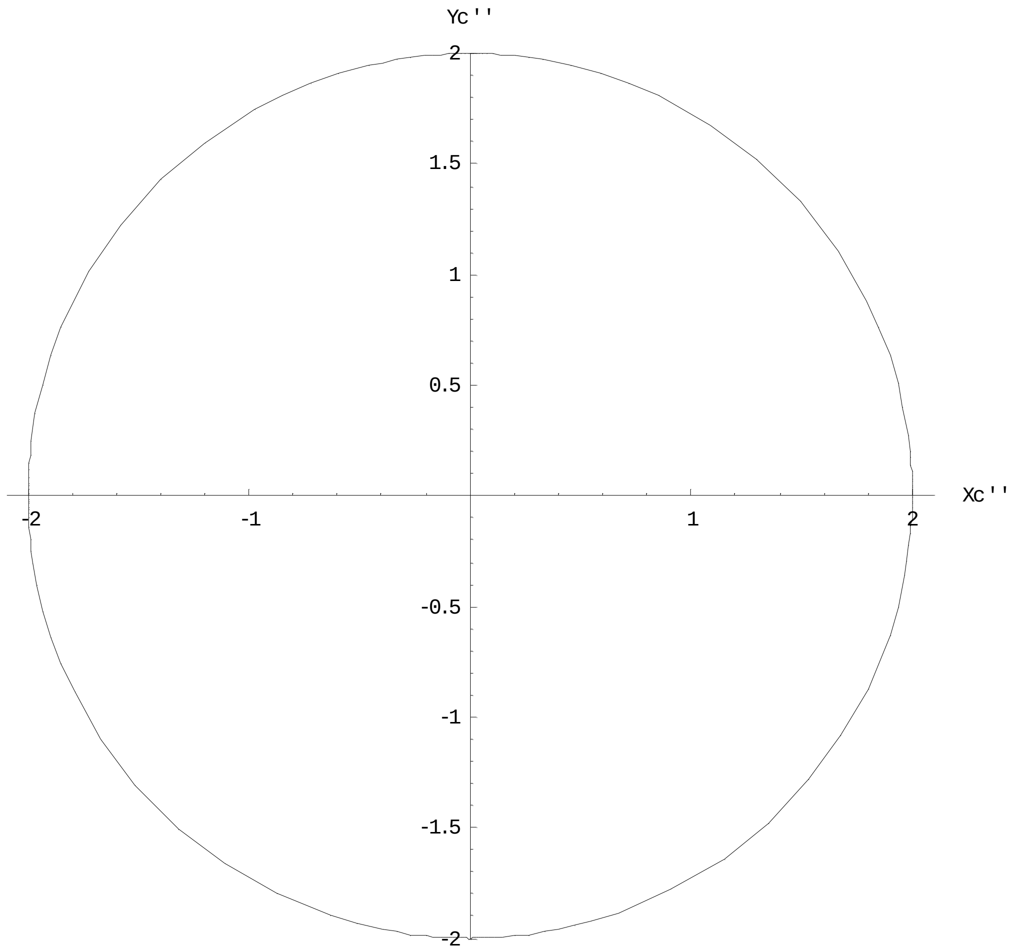


De sleepsnelheid : $\begin{cases} Xs' = -\omega \sin \omega t \\ Ys' = \omega \cos \omega t \end{cases}$ en sleepversnelling voor AP=1 : $\begin{cases} Xs'' = -\omega^2 \cos \omega t \\ Ys'' = -\omega^2 \sin \omega t \end{cases}$



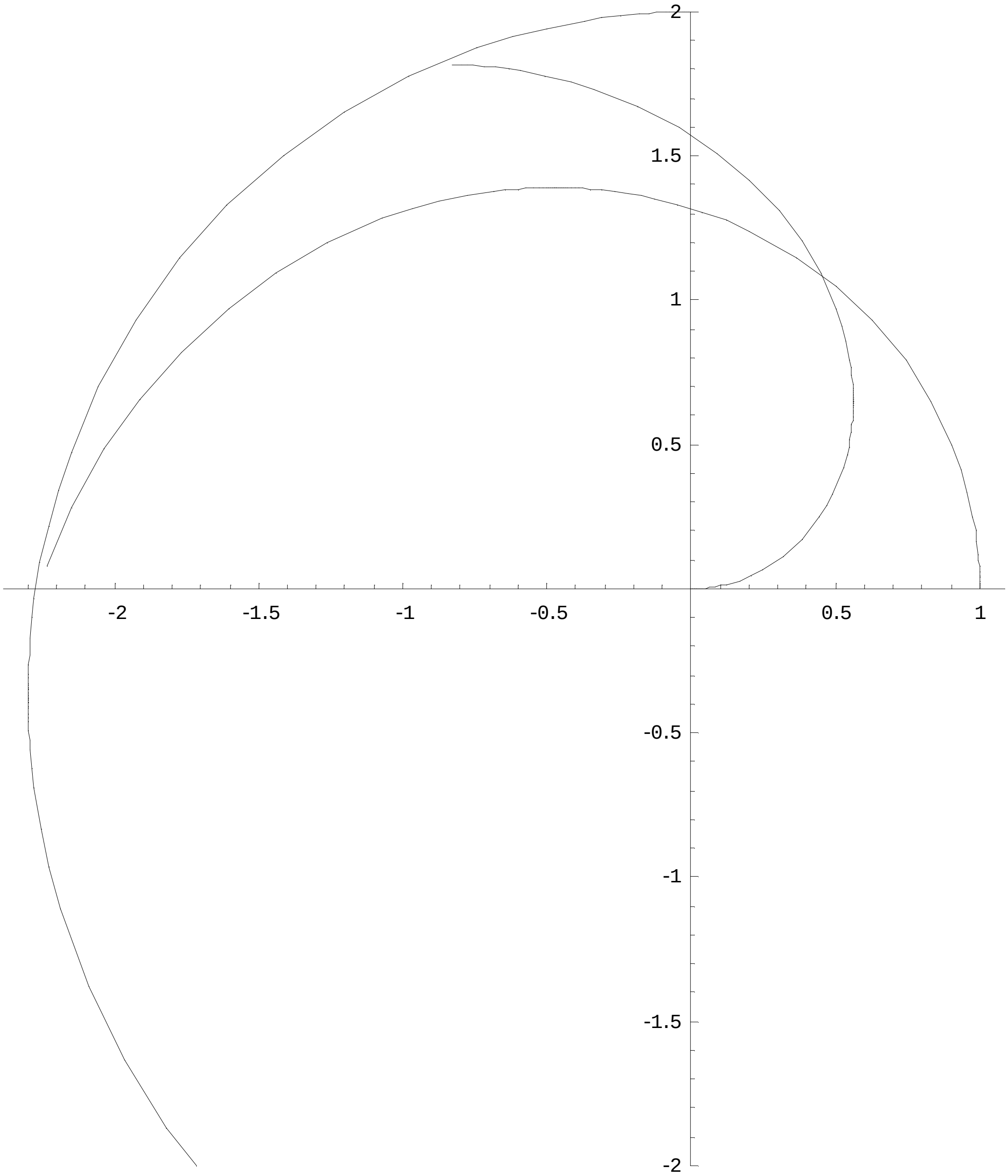


De Coriolis versnelling : $\begin{cases} Xc'' = -2v_0 \omega \sin \omega t \\ Yc'' = 2v_0 \omega \cos \omega t \end{cases}$

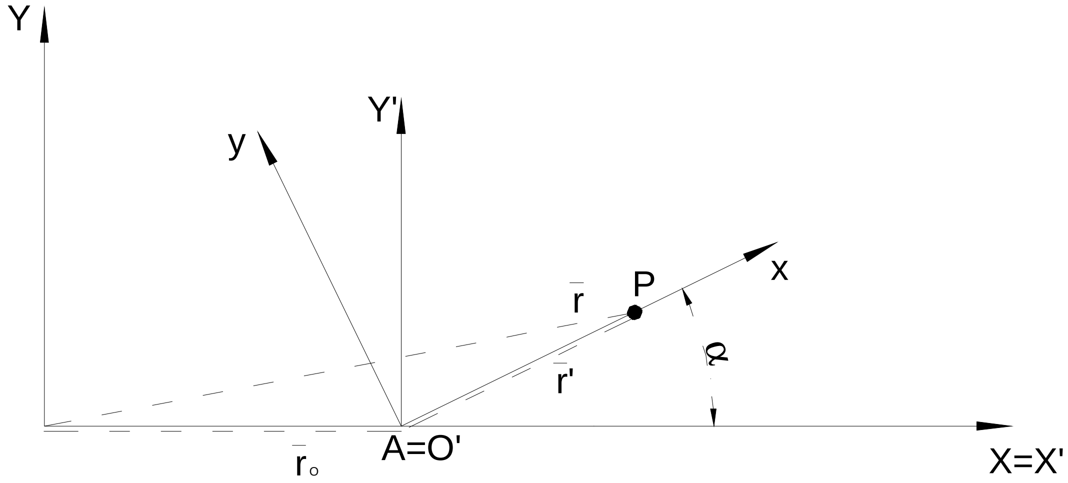


Overzicht :

$$\overline{\mathbf{a}} = \overline{\mathbf{a}}' + \dot{\overline{\varpi}} \times \overline{\mathbf{r}} + 2\overline{\varpi} \times \overline{\mathbf{v}}' + \overline{\varpi} \times (\overline{\varpi} \times \overline{\mathbf{r}})$$



Oefening 2:



$$\alpha = \omega t \qquad\qquad AP = L\cos\frac{t}{2}$$

Relatieve baan (t.o.v. x, y):

$$\bar{r}'\begin{cases} x = L\cos\dfrac{t}{2} \\ y = 0 \end{cases}$$

Absolute baan (t.o.v. XY):

$$\bar{r} = \bar{r}_o + \bar{r}'$$

$$\textbf{met} \quad \bar{r}_o\begin{cases} X_o = v_o t \\ Y_o = 0 \end{cases}$$

$$\bar{r}'\begin{cases} X' = L\cos\dfrac{t}{2}.\cos\omega t \\ Y' = L\cos\dfrac{t}{2}.\sin\omega t \end{cases}$$

dus de absolute baan wordt :

$$\bar{r}\begin{cases} X = v_o t + L\cos\dfrac{t}{2}.\cos\omega t \\ Y = L\cos\dfrac{t}{2}.\sin\omega t \end{cases}$$

Sleepbaan : punt P vasthouden in relatieve assen op t_1

$$\begin{cases} X_S = v_o t + L\cos\dfrac{t_1}{2}.\cos\omega t \\ Y_S = L\cos\dfrac{t_1}{2}.\sin\omega t \end{cases}$$

a)

relatieve snelheid $\bar{v}_{r'} = \dot{\bar{r}}'$

$$\bar{v}_{r'}\begin{cases} \dot{x} = -\dfrac{L}{2}\sin\dfrac{t}{2} \\ \dot{y} = 0 \end{cases} \qquad \text{t.o.v. xy}$$

$$\bar{v}_{r'}\begin{cases} \dot{X} = -\dfrac{L}{2}\sin\dfrac{t}{2}.\cos\omega t \\ \dot{Y} = -\dfrac{L}{2}\sin\dfrac{t}{2}.\sin\omega t \end{cases} \qquad \text{t.o.v. XY}$$

sleepsnelheid afgeleide van de sleepbaan met t_1=c^{te}

$$\begin{cases} \dot{X}_S = v_o - L\omega\cos\dfrac{t_1}{2}.\sin\omega t \\ \dot{Y}_S = L\omega\cos\dfrac{t_1}{2}.\cos\omega t \end{cases} \qquad \text{t.o.v. XY}$$

absolute snelheid

$$\bar{v} = \bar{v}_{r'} + \bar{v}_S$$

$$\bar{v}\begin{cases} \dot{X} = -\dfrac{L}{2}\sin\dfrac{t}{2}.\cos\omega t + v_o - L\omega\cos\dfrac{t}{2}.\sin\omega t \\ \dot{Y} = -\dfrac{L}{2}\sin\dfrac{t}{2}.\sin\omega t - L\omega\cos\dfrac{t}{2}.\cos\omega t \end{cases} \qquad \text{t.o.v. XY}$$

of absolute baan afleiden

b)
<u>relatieve versnelling</u>

$$\bar{a}_{r'}\begin{cases}\ddot{x} = -\dfrac{L}{4}\cos\dfrac{t}{2} \\ \ddot{y} = 0\end{cases} \qquad \text{t.o.v. xy}$$

$$\bar{a}_{r'}\begin{cases}\ddot{X} = -\dfrac{L}{4}\cos\dfrac{t}{2}.\cos\omega t \\ \ddot{Y} = -\dfrac{L}{4}\cos\dfrac{t}{2}.\sin\omega t\end{cases} \qquad \text{t.o.v. XY}$$

<u>sleepversnelling</u> $\bar{a}_S = \dot{\bar{v}}_S$ met $t_1 = c^{te}$

$$\bar{a}_S\begin{cases}\ddot{X}_S = -L\omega^2\cos\dfrac{t_1}{2}.\cos\omega t \\ \ddot{Y}_S = -L\omega^2\cos\dfrac{t_1}{2}.\sin\omega t\end{cases} \qquad \text{t.o.v. XY}$$

<u>absolute versnelling</u> afleiding van absolute snelheid

$$\bar{a}\begin{cases}\ddot{X} = -\dfrac{L}{4}\cos\dfrac{t}{2}.\cos\omega t + \dfrac{L}{2}\omega\sin\dfrac{t}{2}.\sin\omega t + \dfrac{L}{2}\omega\sin\dfrac{t}{2}.\sin\omega t - L\omega^2\cos\dfrac{t}{2}.\cos\omega t \\ \ddot{Y} = -\dfrac{L}{4}\cos\dfrac{t}{2}.\sin\omega t - \dfrac{L}{2}\omega\sin\dfrac{t}{2}.\cos\omega t - \dfrac{L}{2}\omega\sin\dfrac{t}{2}.\cos\omega t - L\omega^2\cos\dfrac{t}{2}.\sin\omega t\end{cases}$$

$$\bar{a}\begin{cases}\ddot{X} = -\dfrac{L}{4}\cos\dfrac{t}{2}.\cos\omega t + L\omega\sin\dfrac{t}{2}.\sin\omega t - L\omega^2\cos\dfrac{t}{2}.\cos\omega t \\ \ddot{Y} = -\dfrac{L}{4}\cos\dfrac{t}{2}.\sin\omega t - L\omega\sin\dfrac{t}{2}.\cos\omega t - L\omega^2\cos\dfrac{t}{2}.\sin\omega t\end{cases}$$

<u>coriolisversnelling</u> $\bar{a}_C = \bar{a} - \bar{a}_S - \bar{a}_r$

$$\bar{a}_C\begin{cases}\ddot{X}_C = L\omega\sin\dfrac{t}{2}.\sin\omega t \\ \ddot{Y}_C = -L\omega\sin\dfrac{t}{2}.\cos\omega t\end{cases}$$

Werken met vectoriële produkten :

$$\overline{v} = \overline{v}_o + \overline{v}' + \overline{\omega} \times \overline{r}'$$
$$\overline{a} = \overline{a}_o + \overline{a}' + \dot{\overline{\omega}} \times \overline{r}' + \overline{\omega} \times (\overline{\omega} \times \overline{r}') + 2\overline{\omega} \times \overline{v}'$$

Opgelet alle vectoren moeten in dezelfde assen uitgedrukt worden !!

$$\begin{aligned}
\overline{r}' \quad &= L\cos\frac{t}{2}\,\overline{1}_x + 0\,\overline{1}_y \\
&= L\cos\frac{t}{2}\,(\cos\omega t\,\overline{1}_X + \sin\omega t\,\overline{1}_Y) \\
&= L\cos\frac{t}{2}\,.\cos\omega t\;\overline{1}_X + L\cos\frac{t}{2}.\sin\omega t\;\overline{1}_Y
\end{aligned}$$

$$\begin{aligned}
\overline{v}' \quad &= -\frac{L}{2}\sin\frac{t}{2}\,\overline{1}_x + 0\,\overline{1}_y \\
&= -\frac{L}{2}\sin\frac{t}{2}\,(\cos\omega t\,\overline{1}_X + \sin\omega t\,\overline{1}_Y) \\
&= -\frac{L}{2}\sin\frac{t}{2}\,.\cos\omega t\;\overline{1}_X - \frac{L}{2}\sin\frac{t}{2}.\sin\omega t\;\overline{1}_Y
\end{aligned}$$

$$\overline{a}' = -\frac{L}{4}\cos\frac{t}{2}.\cos\omega t\,\overline{1}_X - \frac{L}{4}\cos\frac{t}{2}.\sin\omega t\,\overline{1}_Y$$

$$\overline{v}_o = v_o\,\overline{1}_X$$

$$\overline{\omega} = \omega\,\overline{1}_Z$$

$$\Rightarrow \quad \overline{v} = -\frac{L}{2}\sin\frac{t}{2}.\cos\omega t\,\overline{1}_X - \frac{L}{2}\sin\frac{t}{2}.\sin\omega t\;\overline{1}_Y$$
$$+\begin{vmatrix} \overline{1}_X & \overline{1}_Y & \overline{1}_Z \\ 0 & 0 & \omega \\ L\cos\frac{t}{2}.\cos\omega t & L\cos\frac{t}{2}.\sin\omega t & 0 \end{vmatrix} + v_o\,\overline{1}_X$$

$$\bar{v} = \left(-\frac{L}{2}\sin\frac{t}{2}.\cos\omega t - L\omega\cos\frac{t}{2}.\sin\omega t + v_o\right)\bar{1}_X$$

$$+\left(-\frac{L}{2}\sin\frac{t}{2}.\sin\omega t + L\omega\cos\frac{t}{2}.\cos\omega t\right)\bar{1}_Y$$

$$\bar{a} = 0 - \frac{L}{4}\cos\frac{t}{2}.\cos\omega t\,\bar{1}_X - \frac{L}{4}\cos\frac{t}{2}.\sin\omega t\,\bar{1}_Y$$

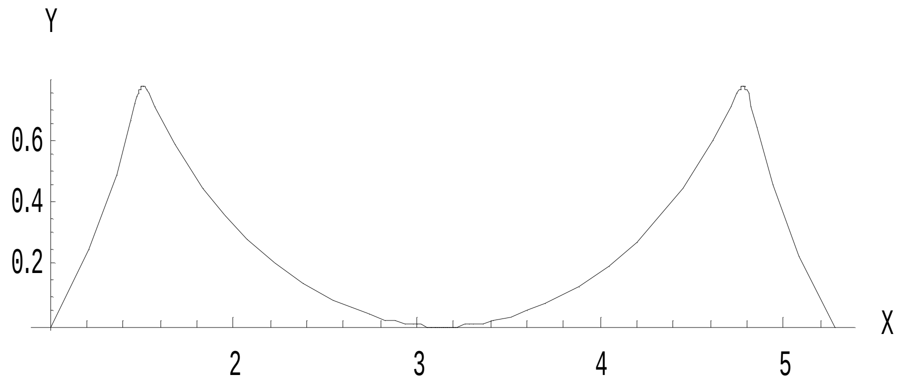
$$+\begin{vmatrix} \bar{1}_X & \bar{1}_Y & \bar{1}_Z \\ 0 & 0 & \omega \\ -L\omega\cos\frac{t}{2}.\sin\omega t & L\omega\cos\frac{t}{2}.\cos\omega t & 0 \end{vmatrix}$$

$$+2\begin{vmatrix} \bar{1}_X & \bar{1}_Y & \bar{1}_Z \\ 0 & 0 & \omega \\ -\frac{L}{2}\sin\frac{t}{2}.\cos\omega t & -\frac{L}{2}\sin\frac{t}{2}.\sin\omega t & 0 \end{vmatrix}$$

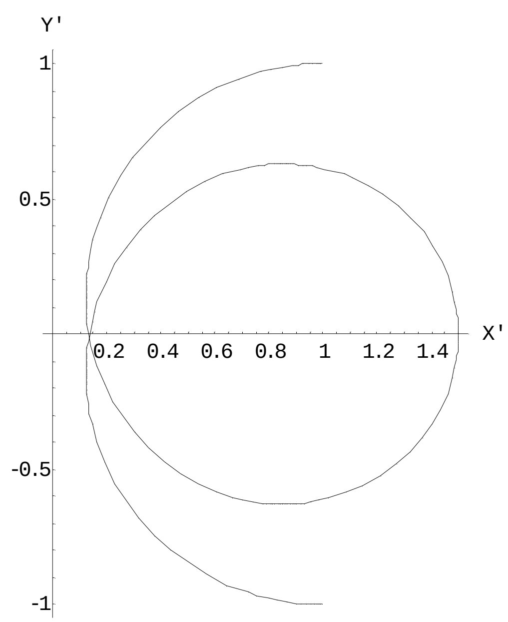
$$\bar{a} = \left(-\frac{L}{4}\cos\frac{t}{2}.\cos\omega t - L\omega^2\cos\frac{t}{2}.\cos\omega t + L\omega\sin\frac{t}{2}.\sin\omega t\right)\bar{1}_X$$

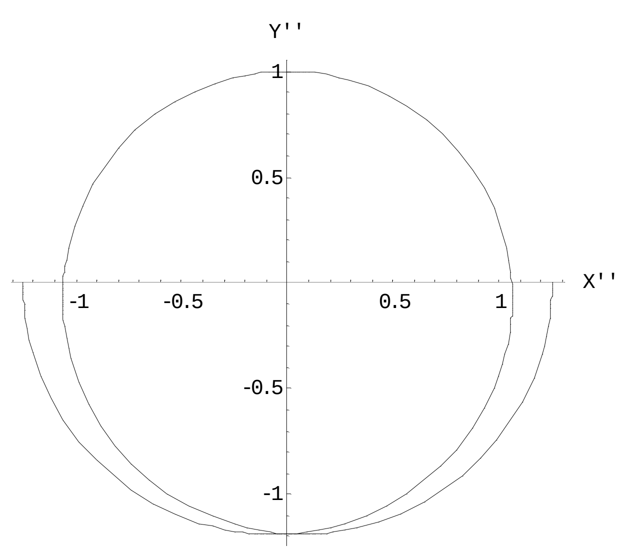
$$+\left(-\frac{L}{4}\cos\frac{t}{2}.\sin\omega t - L\omega^2\cos\frac{t}{2}.\sin\omega t - L\omega\sin\frac{t}{2}.\cos\omega t\right)\bar{1}_Y$$

Absolute baan

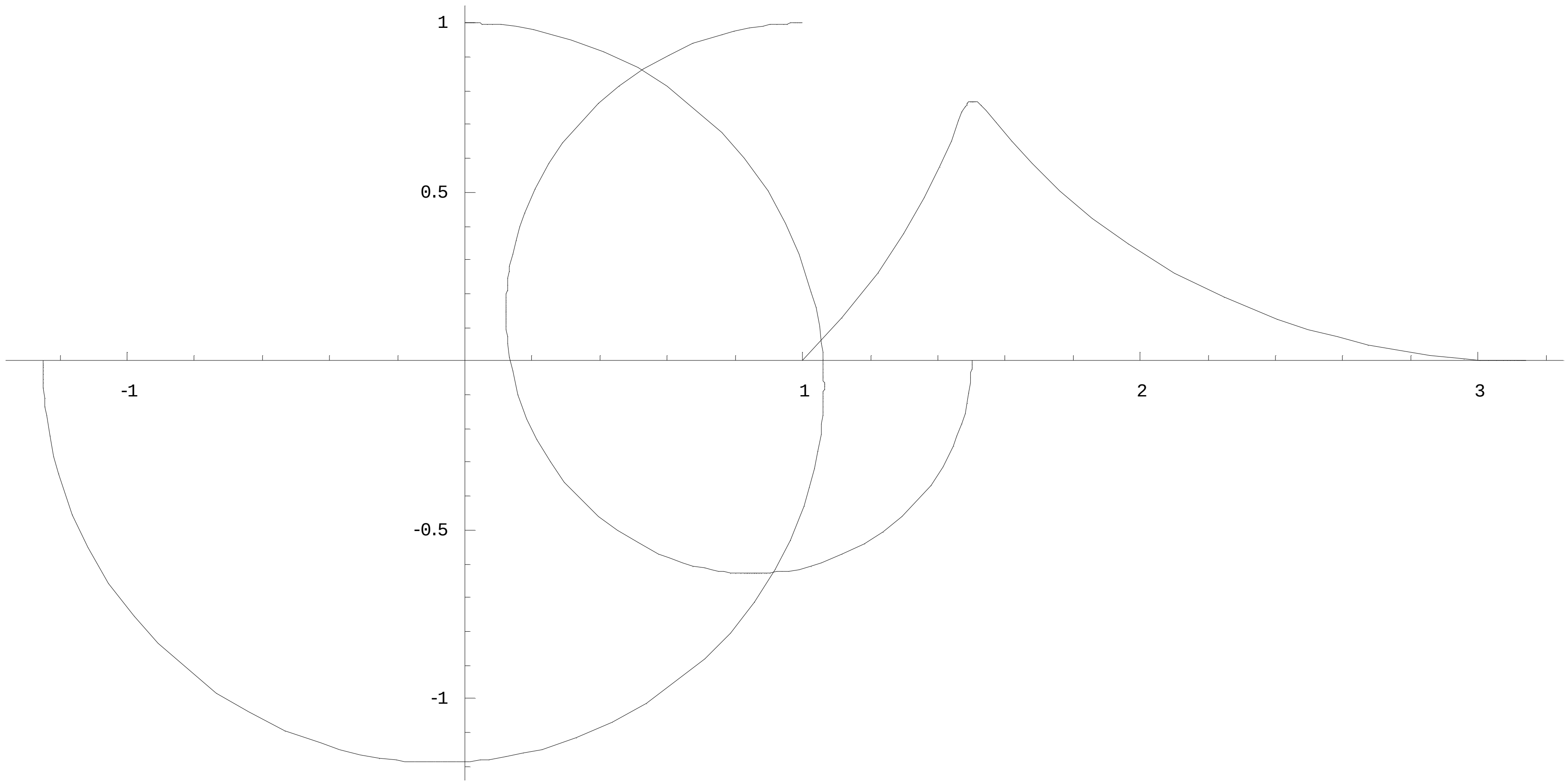


Absolute snelheid en versnelling

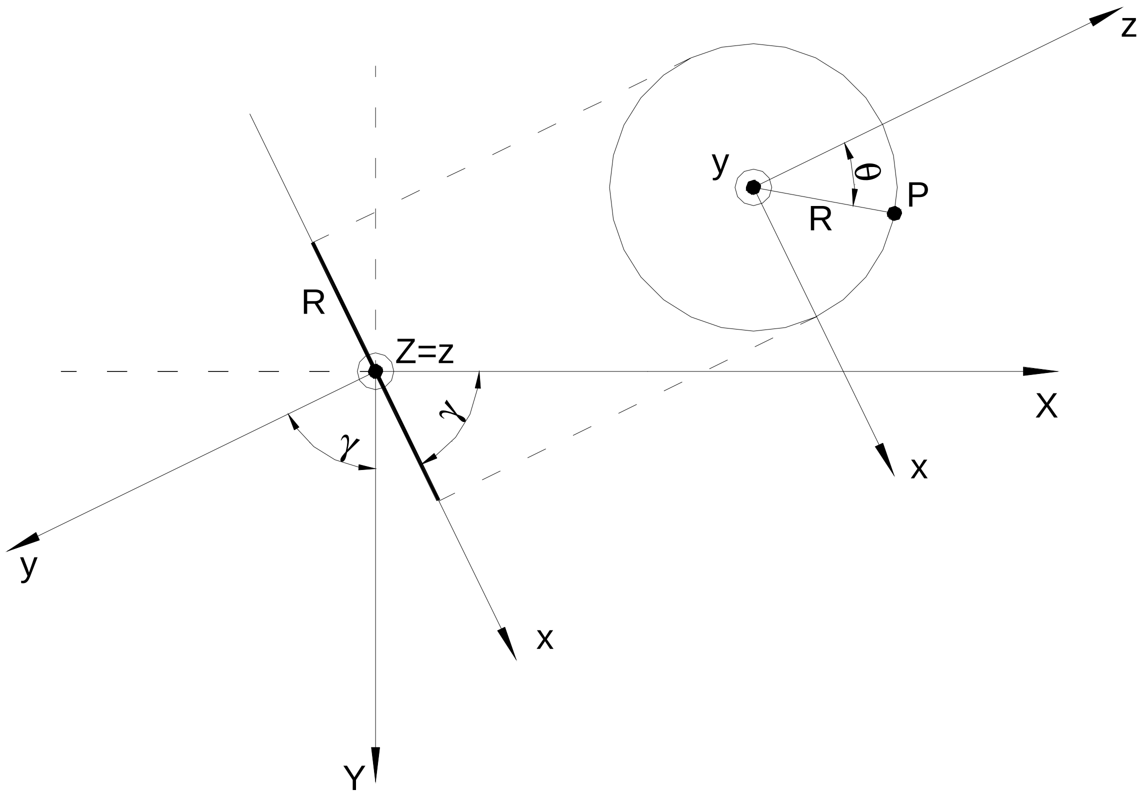




Overzicht



Oefening 10:



$$\theta = \gamma = \omega_o t \qquad \textbf{en} \qquad \overline{\omega}_o(0,0,\omega_o) \text{ in absolute assen XYZ}$$

a)

relatieve baan:

$$\overline{\mathbf{r}}' \begin{cases} \mathbf{x = R\sin\theta = R\sin\omega_o t} \\ \mathbf{y = 0} \\ \mathbf{z = R\cos\theta = R\cos\omega_o t} \end{cases} \qquad \text{in xyz}$$

$$\overline{\mathbf{1}}_x = \cos\gamma\,\overline{\mathbf{1}}_X + \sin\gamma\,\overline{\mathbf{1}}_Y \qquad \gamma = \omega_o t$$

en $$(\overline{\mathbf{1}}_y = -\sin\gamma\,\overline{\mathbf{1}}_X + \cos\gamma\,\overline{\mathbf{1}}_Y)$$

$$\overline{\mathbf{1}}_z = \overline{\mathbf{1}}_Z$$

absolute baan:

$$\bar{r}\begin{cases} X = R\sin\omega_o t.\cos\omega_o t = \frac{R}{2}\sin 2\omega_o t \\ Y = R\sin\omega_o t.\sin\omega_o t = R\sin^2\omega_o t \\ Z = R\cos\omega_o t \end{cases} \qquad \text{in XYZ}$$

sleepbaan: punt P vast in relatieve assen op $t=t_1$

$$\bar{r}\begin{cases} X = R\sin\omega_o t_1.\cos\omega_o t \\ Y = R\sin\omega_o t_1.\sin\omega_o t \\ Z = R\cos\omega_o t_1 \end{cases} \qquad \text{in XYZ}$$

b)

relatieve snelheid:

$$\bar{v}'\begin{cases} \dot{x} = R\omega_o\cos\omega_o t \\ \dot{y} = 0 \\ \dot{z} = -R\omega_o\sin\omega_o t \end{cases} \qquad \text{in xyz}$$

$$\bar{v}'\begin{cases} \dot{X} = R\omega_o\cos\omega_o t.\cos\omega_o t \\ \dot{Y} = R\omega_o\cos\omega_o t.\sin\omega_o t \\ \dot{Z} = -R\omega_o\sin\omega_o t \end{cases} \qquad \text{in XYZ}$$

sleepsnelheid: afleiden van de sleepbaan met $t_1=c^{te}$

$$\bar{v}_S\begin{cases} \dot{X}_S = -R\omega_o\sin\omega_o t_1.\sin\omega_o t \\ \dot{Y}_S = R\omega_o\sin\omega_o t_1.\cos\omega_o t \\ \dot{Z}_S = 0 \end{cases} \qquad \text{in XYZ}$$

absolute snelheid: $\mathbf{\bar{v} = \bar{v}' + \bar{v}_S}$ **of afleiden van** $\mathbf{\bar{r}}$

$$\bar{v}_a \begin{cases} \dot{X} = R\omega_o \cos 2\omega_o t \\ \dot{Y} = 2R\omega_o \sin\omega_o t.\cos\omega_o t = R\omega_o \sin 2\omega_o t \\ \dot{Z} = -R\omega_o \sin\omega_o t \end{cases} \qquad \text{inXYZ}$$

c)

relatieve versnelling:

$$\bar{a}' \begin{cases} \ddot{x} = -R\omega_o^2 \sin\omega_o t \\ \ddot{y} = 0 \\ \ddot{z} = -R\omega_o^2 \cos\omega_o t \end{cases} \qquad \text{in xyz}$$

$$\bar{a}' \begin{cases} \ddot{X} = -R\omega_o^2 \sin\omega_o t.\cos\omega_o t \\ \ddot{Y} = -R\omega_o^2 \sin\omega_o t.\sin\omega_o t \\ \ddot{Z} = -R\omega_o^2 \cos\omega_o t \end{cases} \qquad \text{in XYZ}$$

sleepversnelling:

$$\bar{a}_S \begin{cases} \ddot{X}_S = -R\omega_o^2 \sin\omega_o t_1.\cos\omega_o t \\ \ddot{Y}_S = -R\omega_o^2 \sin\omega_o t_1.\sin\omega_o t \\ \ddot{Z}_S = 0 \end{cases} \qquad \text{in XYZ}$$

coriolisversnelling:

$$\bar{a}_C = 2\bar{\omega}_o \,x\, \bar{v}' = 2\begin{vmatrix} \bar{1}_X & \bar{1}_Y & \bar{1}_Z \\ 0 & 0 & \omega_o \\ R\omega_o \cos^2\omega_o t & \dfrac{R\omega_o \sin 2\omega_o t}{2} & -R\omega_o \sin\omega_o t \end{vmatrix}$$

$$\bar{a}_C = \begin{cases} \ddot{X}_C = -R\omega_o^2 \sin 2\omega_o t \\ \ddot{Y}_C = 2R\omega_o^2 \cos^2\omega_o t \\ \ddot{Z}_C = 0 \end{cases} \qquad \text{in XYZ}$$

absolute versnelling:

$$\bar{\mathbf{a}} = \begin{cases} \ddot{\mathbf{X}} = -2\mathbf{R}\omega_o^2 \sin 2\omega_o t \\ \ddot{\mathbf{Y}} = 2\mathbf{R}\omega_o^2 \cos 2\omega_o t \\ \ddot{\mathbf{Z}} = -\mathbf{R}\omega_o^2 \cos \omega_o t \end{cases} \qquad \text{in XYZ}$$